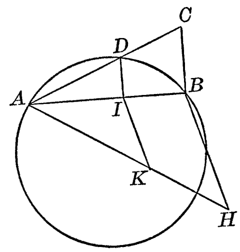
SOLUTION BY EMMA M. GIBSON, Drury College.

The ruler and compasses are sufficient to construct a triangle whose hypotenuse shall be divided in any ratio as $m : n$, according to the conditions of the problem.



For let AB be the given chord. From A draw any line as AH and from A lay off $AK = m$ and $KH = n$. Then connect B and H and draw KI parallel to HB and intersecting AB in I. At I erect a perpendicular to AB intersecting the circle in D. Join A and D by a straight line and produce it until it meets a perpendicular erected at B in C. Then the right triangle ABC has its hypotenuse, AC, divided at D in the ratio of $m : n$.

For in the similar triangles AIK and ABH, $AI : IB = m : n$ and in the similar triangles ABC and AID, $AI : IB = AD : DC$. Hence, $AD : DC = m : n$.

When $m = n$, D bisects the hypotenuse.

Also solved by CLIFFORD N. MILLS, F. M. MORGAN, C. HORNUNG, HORACE OLSON, and S. W. REAVES.

429. Proposed by JOHN A. BIGBEE, Little Rock, Ark.

In the trihedral angle V-ABC, the face angle AVB is bisected by the straight line VD. Is it true that the angle DVC is less than, equal to, or greater than, half the sum of the angles AVC and BVC, according as $\angle\ CVD$ is less than, equal to, or greater than 90°?

SOLUTION BY A. M. HARDING, University of Arkansas.

(1) $\angle\ CVD < 90°$. (Fig. 1.) Take $VA = VB$. Join A and B. Let this line cut the bisector of $\angle\ AVB$ at D. Through D pass a plane perpendicular to

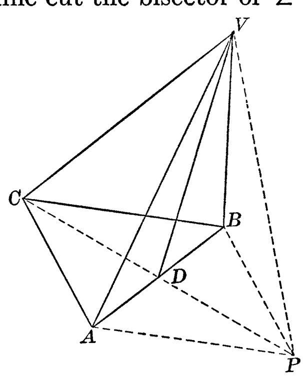


FIG. 1.

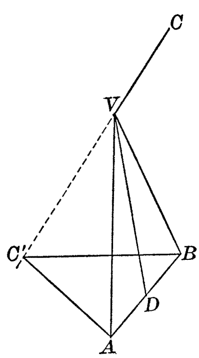


FIG. 2.

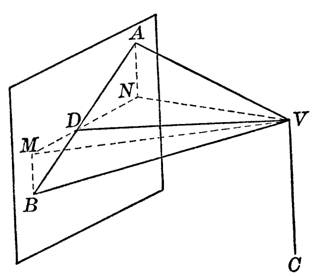


FIG. 3.

VD cutting VC at C. Produce CD to P making $DC = DP$. Draw the lines AP, BP, and VP. From this construction it easily follows that $\angle BVC = \angle PVA$ and $\angle CVP = 2\angle CVD$. But $\angle CVP < \angle AVC + \angle PVA$. Hence $\angle CVD < \frac{1}{2}(\angle AVC + \angle BVC)$.

(2) $\angle CVD > 90°$. (Fig. 2.) Produce CV through V and draw plane ABC' as in (1). Then $2\angle C'VD < \angle AVC' + \angle BVC'$ by (1). Hence $2(180° - \angle C'VD) > 180° - \angle AVC' + 180° - \angle BVC'$ or $\angle CVD > \frac{1}{2}(\angle AVC + \angle BVC)$.

(3) $\angle CVD = 90°$. (Fig. 3.) In this case the plane through AB is parallel to VC. Draw a plane MNV through VD perpendicular to VC and cut by planes BVC and AVC in the lines MV and NV respectively. It can be easily shown that $\angle MVB = \angle NVA$. Hence

$$\angle AVC + \angle BVC = 180°$$

and

$$\angle CVD = \tfrac{1}{2}(\angle AVC + \angle BVC).$$

Also solved by B. Libby, F. M. Morgan, and Geo. W. Hartwell.

CALCULUS.

338. Proposed by RICHARD LOCHNER, Philadelphia, Pa.

An elliptical field has a major axis of 100 feet and a minor axis of 10 feet. A cow is tethered at the end of the major axis and another at the end of the minor axis. If each cow can graze over half the field, how long is the rope of each? What is the area of the portion over which the cows can graze in common?

Solution by B. F. Finkel, Drury College.

The central equation of the elliptic field is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Let r_1 be the length of rope by which one cow is tethered at the point A_1, the right-hand extremity of the major axis. The equation of the circle over which this cow can browse is $(x - a)^2 + y^2 = r^2$. The coördinates of the point of intersection, P_1, of this circle with the ellipse are

$$(x_1, y_1) \equiv \left(a\frac{[a^2 - \sqrt{b^4 + r^2(a^2 - b^2)}]}{(a^2 - b^2)}, \right.$$
$$\left. \sqrt{2a^4\frac{\sqrt{b^4 + r^2(a^2 - b^2)} - b^2(a^2 - b^2)r^2 - a^2(a^4 + b^4)}{(a^2 - b^2)}} \right),$$

The area common to the ellipse and the circle is

$$\text{area} = 2\int_0^{y_1}\int_{a-\sqrt{r^2-y^2}}^{(a/b)\sqrt{b^2-y^2}} dxdy$$
$$= 2\left[\frac{a}{b}\frac{y_1}{2}\sqrt{b^2 - y_1^2} + \frac{b^2}{2}\sin^{-1}\frac{y'}{b} - ay' + \frac{y'}{2}\sqrt{r^2 - y_1^2} + \frac{r^2}{2}\sin^{-1}\frac{y_1}{r}\right],$$

which, by the conditions of the problem, $= \frac{1}{2}\pi ab$.